SUPERSYMMETRY PHENOMENOLOGY

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This is a very pedagogical review of supersymmetry phenomenology, given at ICTP Summer School in 1999, aimed mostly at students who had never studied supersymmetry before. It starts with an analogy that the reason why supersymmetry is needed is similar to the reason why the positron exists. It introduces the construction of supersymmetric Lagrangians in a practical way. The low-energy constraints, renormalization-group analyses, collider phenomenology, and frameworks of mediating supersymmetry breaking are briefly discussed.

1 Motivation

1.1 Problems in the Standard Model

The Standard Model of particle physics, albeit extremely successful phenomenologically, has been regarded only as a low-energy effective theory of the yet-more-fundamental theory. One can list many reasons why we think this way, but a few are named below.

First of all, the quantum number assignments of the fermions under the

Table 1. The fermionic particle content of the Standard Model. Here we've put primes on the neutrinos in the same spirit of putting primes on the down-quarks in the quark doublets, indicating that the mass eigenstates are rotated by the MNS and CKM matrices, respectively. The subscripts g, r, b refer to colors.

$$
\begin{pmatrix} \nu'_e \\ e \end{pmatrix}^{-1/2}_{L} \quad \begin{pmatrix} \nu'_\mu \\ \mu \end{pmatrix}^{-1/2}_{L} \quad \begin{pmatrix} \nu'_\tau \\ \tau \end{pmatrix}^{-1/2}_{L} \quad e^{-1}_R \quad \mu^{-1}_R \quad \tau^{-1}_R
$$

$$
\begin{pmatrix} u \\ d' \end{pmatrix}^{1/6}_{L,g} \quad \begin{pmatrix} c \\ s' \end{pmatrix}^{1/6}_{L,g} \quad \begin{pmatrix} t \\ b' \end{pmatrix}^{1/6}_{L,g} \quad \begin{matrix} u^{2/3}_{R,g} \\ d^{-1/3}_{R,g} \end{matrix} \quad \begin{matrix} c^{2/3}_{R,g} \\ s^{-1/3}_{R,g} \end{matrix} \quad \begin{matrix} t^{2/3}_{R,g} \\ b^{-1/3}_{R,g} \end{matrix}
$$

$$
\begin{pmatrix} u \\ d' \end{pmatrix}^{1/6}_{L,r} \quad \begin{pmatrix} c \\ s' \end{pmatrix}^{1/6}_{L,r} \quad \begin{pmatrix} t \\ b' \end{pmatrix}^{1/6}_{L,r} \quad \begin{matrix} u^{2/3}_{R,r} \\ d^{-1/3}_{R,r} \end{matrix} \quad \begin{matrix} c^{2/3}_{R,r} \\ s^{-1/3}_{R,r} \end{matrix} \quad \begin{matrix} t^{2/3}_{R,r} \\ b^{-1/3}_{R,r} \end{matrix}
$$

$$
\begin{pmatrix} u \\ d' \end{pmatrix}^{1/6}_{L,b} \quad \begin{pmatrix} c \\ s' \end{pmatrix}^{1/6}_{L,b} \quad \begin{pmatrix} t \\ b' \end{pmatrix}^{1/6}_{L,b} \quad \begin{matrix} u^{2/3}_{R,b} \\ d^{-1/3}_{R,b} \end{matrix} \quad \begin{matrix} c^{2/3}_{R,b} \\ s^{-1/3}_{R,b} \end{matrix} \quad \begin{matrix} t^{2/3}_{R,b} \\ b^{-1/3}_{R,b} \end{matrix}
$$

Table 2. The bosonic particle content of the Standard Model.

$$\begin{array}{ccc}
W^1, W^2, H^+, H^- & \longrightarrow & W^+, W^- \\
W^3, B, \mathrm{Im}(H^0) & \longrightarrow & \gamma, Z \\
g \times 8 & & \\
\mathrm{Re} H^0 & \longrightarrow & H
\end{array}$$

standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group (Table 1) appear utterly bizarre. Probably the hypercharges are the weirdest of all. These assignments, however, are crucial to guarantee the cancellation of anomalies which could jeopardize the gauge invariance at the quantum level, rendering the theory inconsistent. Another related puzzle is why the hypercharges are quantized in the unit of $1/6$. In principle, the hypercharges can be any numbers, even irrational. However, the quantized hypercharges are responsible for neutrality of bulk matter $Q(e) + 2Q(u) + Q(d) = Q(u) + 2Q(d) = 0$ at a precision of 10^{-21}.[1]

The gauge group itself poses a question as well. Why are there seemingly unrelated three independent gauge groups, which somehow conspire together to have anomaly-free particle content in a non-trivial way? Why is "the strong interaction" strong and "the weak interaction" weaker?

The essential ingredient in the Standard Model which appears the ugliest to most people is the electroweak symmetry breaking. In the list of bosons in the Standard Model Table 2, the gauge multiplets are necessary consequences of the gauge theories, and they appear natural. They of course all carry spin 1. However, there is only one spinless multiplet in the Standard Model: the Higgs doublet

$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{1}$$

which condenses in the vacuum due to the Mexican-hat potential. It is introduced just for the purpose of breaking the electroweak symmetry $SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{QED}}$. The potential has to be arranged in a way to break the symmetry without any microscopic explanations.

Why is there a seemingly unnecessary three-fold repetition of "generations"? Even the second generation led the Nobel Laureate I.I. Rabi to ask "who ordered muon?" Now we face even more puzzling question of having three generations. And why do the fermions have a mass spectrum which stretches over almost six orders of magnitude between the electron and the top quark? This question becomes even more serious once we consider the

recent evidence for neutrino oscillations which suggest the mass of the third-generation neutrino ν_τ' of about 0.05 eV.[2] This makes the mass spectrum stretching over *thirteen* orders of magnitude. We have no concrete understanding of the mass spectrum nor the mixing patterns.

1.2 Drive to go to Shorter Distances

All the puzzles raised in the previous section (and more) cry out for a more fundamental theory underlying the Standard Model. What history suggests is that the fundamental theory lies always at shorter distances than the distance scale of the problem. For instance, the equation of state of the ideal gas was found to be a simple consequence of the statistical mechanics of free molecules. The van der Waals equation, which describes the deviation from the ideal one, was the consequence of the finite size of molecules and their interactions. Mendeleev's periodic table of chemical elements was understood in terms of the bound electronic states, Pauli exclusion principle and spin. The existence of varieties of nuclide was due to the composite nature of nuclei made of protons and neutrons. The list would go on and on. Indeed, seeking answers at more and more fundamental level is the heart of the physical science, namely the reductionist approach.

The distance scale of the Standard Model is given by the size of the Higgs boson condensate $v = 250$ GeV. In natural units, it gives the distance scale of $d = \hbar c/v = 0.8 \times 10^{-16}$ cm. We therefore would like to study physics at distance scales shorter than this eventually, and try to answer puzzles whose partial list was given in the previous section.

Then the idea must be that we imagine the Standard Model to be valid down to a distance scale shorter than d, and then new physics will appear which will take over the Standard Model. But applying the Standard Model to a distance scale shorter than d poses a serious theoretical problem. In order to make this point clear, we first describe a related problem in the classical electromagnetism, and then discuss the case of the Standard Model later along the same line.[3]

1.3 Positron Analogue

In the classical electromagnetism, the only dynamical degrees of freedom are electrons, electric fields, and magnetic fields. When an electron is present in the vacuum, there is a Coulomb electric field around it, which has the energy

of

$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_e}. \tag{2}$$

Here, r_e is the "size" of the electron introduced to cutoff the divergent Coulomb self-energy. Since this Coulomb self-energy is there for every electron, it has to be considered to be a part of the electron rest energy. Therefore, the mass of the electron receives an additional contribution due to the Coulomb self-energy:

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} + \Delta E_{\text{Coulomb}}. \tag{3}$$

Experimentally, we know that the "size" of the electron is small, $r_e \lesssim 10^{-17}$ cm. This implies that the self-energy ΔE is greater than 10 GeV or so, and hence the "bare" electron mass must be negative to obtain the observed mass of the electron, with a fine cancellation like

$$0.511 = -9999.489 + 10000.000 \text{MeV}. \tag{4}$$

Even setting a conceptual problem with a negative mass electron aside, such a fine-cancellation between the "bare" mass of the electron and the Coulomb self-energy appears ridiculous. In order for such a cancellation to be absent, we conclude that the classical electromagnetism cannot be applied to distance scales shorter than $e^2/(4\pi\varepsilon_0 m_e c^2) = 2.8 \times 10^{-13}$ cm. This is a long distance in the present-day particle physics' standard.

The resolution to the problem came from the discovery of the anti-particle of the electron, the positron, or in other words by doubling the degrees of freedom in the theory. The Coulomb self-energy discussed above can be depicted by a diagram where the electron emits the Coulomb field (a virtual photon) which is absorbed later by the electron (the electron "feels" its own Coulomb field). But now that the positron exists (thanks to Anderson back in 1932), and we also know that the world is quantum mechanical, one should think about the fluctuation of the "vacuum" where the vacuum produces a pair of an electron and a positron out of nothing together with a photon, within the time allowed by the energy-time uncertainty principle $\Delta t \sim \hbar/\Delta E \sim \hbar/(2m_e c^2)$. This is a new phenomenon which didn't exist in the classical electrodynamics, and modifies physics below the distance scale $d \sim c\Delta t \sim \hbar c/(2m_e c^2) = 200 \times 10^{-13}$ cm. Therefore, the classical electrodynamics actually did have a finite applicability only down to this distance scale, much earlier than 2.8×10^{-13} cm as exhibited by the problem of the fine cancellation above. Given this vacuum fluctuation process, one should also consider a process where the electron sitting in the vacuum by chance

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annihilates with the positron and the photon in the vacuum fluctuation, and the electron which used to be a part of the fluctuation remains instead as a real electron. V. Weisskopf[4] calculated this contribution to the electron self-energy for the first time, and found that it is negative and cancels the leading piece in the Coulomb self-energy exactly:

$$\Delta E_{\text{pair}} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}. \tag{5}$$

After the linearly divergent piece $1/r_e$ is canceled, the leading contribution in the $r_e \rightarrow 0$ limit is given by

$$\Delta E = \Delta E_{\text{Coulomb}} + \Delta E_{\text{pair}} = \frac{3\alpha}{4\pi}m_e c^2 \log\frac{\hbar}{m_e c r_e}. \tag{6}$$

There are two important things to be said about this formula. First, the correction ΔE is proportional to the electron mass and hence the total mass is proportional to the "bare" mass of the electron,

$$(m_e c^2)_{obs} = (m_e c^2)_{bare}\left[1 + \frac{3\alpha}{4\pi}\log\frac{\hbar}{m_e c r_e}\right]. \tag{7}$$

Therefore, we are talking about the "percentage" of the correction, rather than a huge additive constant. Second, the correction depends only logarithmically on the "size" of the electron. As a result, the correction is only a 9% increase in the mass even for an electron as small as the Planck distance $r_e = 1/M_{Pl} = 1.6 \times 10^{-33}$ cm.

The fact that the correction is proportional to the "bare" mass is a consequence of a new symmetry present in the theory with the antiparticle (the positron): the chiral symmetry. In the limit of the exact chiral symmetry, the electron is massless and the symmetry protects the electron from acquiring a mass from self-energy corrections. The finite mass of the electron breaks the chiral symmetry explicitly, and because the self-energy correction should vanish in the chiral symmetric limit (zero mass electron), the correction is proportional to the electron mass. Therefore, the doubling of the degrees of freedom and the cancellation of the power divergences lead to a sensible theory of electron applicable to very short distance scales.

1.4 Supersymmetry

In the Standard Model, the Higgs potential is given by

$$V = \mu^2|H|^2 + \lambda|H|^4, \tag{8}$$

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where $v^2 = \langle H \rangle^2 = -\mu^2/2\lambda = (176 \text{ GeV})^2$. Because perturbative unitarity requires that $\lambda \lesssim 1$, $-\mu^2$ is of the order of $(100 \text{ GeV})^2$. However, the mass squared parameter μ^2 of the Higgs doublet receives a quadratically divergent contribution from its self-energy corrections. For instance, the process where the Higgs doublets splits into a pair of top quarks and come back to the Higgs boson gives the self-energy correction

$$\Delta\mu^2_{\text{top}} = -6\frac{h_t^2}{4\pi^2}\frac{1}{r_H^2}, \tag{9}$$

where r_H is the "size" of the Higgs boson, and $h_t \approx 1$ is the top quark Yukawa coupling. Based on the same argument in the previous section, this makes the Standard Model not applicable below the distance scale of 10^{-17} cm.

The motivation for supersymmetry is to make the Standard Model applicable to much shorter distances so that we can hope that answers to many of the puzzles in the Standard Model can be given by physics at shorter distance scales.[5] In order to do so, supersymmetry repeats what history did with the positron: doubling the degrees of freedom with an explicitly broken new symmetry. Then the top quark would have a superpartner, stop,[a] whose loop diagram gives another contribution to the Higgs boson self energy

$$\Delta\mu^2_{\text{stop}} = +6\frac{h_t^2}{4\pi^2}\frac{1}{r_H^2}. \tag{10}$$

The leading pieces in $1/r_H$ cancel between the top and stop contributions, and one obtains the correction to be

$$\Delta\mu^2_{\text{top}} + \Delta\mu^2_{\text{top}} = -6\frac{h_t^2}{4\pi^2}(m_{\tilde{t}}^2 - m_t^2)\log\frac{1}{r_H^2 m_{\tilde{t}}^2}. \tag{11}$$

One important difference from the positron case, however, is that the mass of the stop, $m_{\tilde{t}}$, is unknown. In order for the $\Delta\mu^2$ to be of the same order of magnitude as the tree-level value $\mu^2 = -2\lambda v^2$, we need $m_{\tilde{t}}^2$ to be not too far above the electroweak scale. Similar arguments apply to masses of other superpartners that couple directly to the Higgs doublet. This is the so-called naturalness constraint on the superparticle masses (for more quantitative discussions, see papers[6]).

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[a]This is a terrible name, which was originally meant to be "scalar top." If supersymmetry will be discovered by the next generation collider experiments, we should seriously look for better names for the superparticles.

Of course, supersymmetry is not the only solution discussed in the literature to avoid miraculously fine cancellations in the Higgs boson mass-squared term. Technicolor (see a review[7]) is a beautiful idea which replaces the Higgs doublet by a composite techni-quark condensate. Then $r_H \sim 1$ TeV is a truly physical size of the Higgs doublet and there is no need for fine cancellations. Despite the beauty of the idea, this direction has had problems with generating fermion masses, especially the top quark mass, in a way consistent with the constraints from the flavor-changing neutral currents. The difficulties in the model building, however, do not necessarily mean that the idea itself is wrong; indeed still efforts are being devoted to construct realistic models.

Another recent idea is to lower the Planck scale down to the TeV scale by employing large extra spatial dimensions.[8] This is a new direction which has just started, and there is an intensive activity to find constraints on the idea as well as on model building. Since the field is still new, there is no "standard" framework one can discuss at this point, but this is no surprise given the fact that supersymmetry is still evolving even after almost two decades of intense research.

One important remark about all these ideas is that they inevitably predict interesting signals at TeV-scale collider experiments. While we only discuss supersymmetry in this lecture, it is likely that nature has a surprise ready for us; maybe none of the ideas discussed so far is right. Still we know that there is *something* out there to be uncovered at TeV scale energies.

2 Supersymmetric Lagrangian

We do not go into full-fledged formalism of supersymmetric Lagrangians in this lecture but rather confine ourselves to a practical introduction of how to write down Lagrangians with explicitly broken supersymmetry which still fulfill the motivation for supersymmetry discussed in the previous section. One can find useful discussions as well as an extensive list of references in a nice review by Steve Martin.[9]

2.1 Supermultiplets

Supersymmetry is a symmetry between bosons and fermions, and hence necessarily relates particles with different spins. All particles in supersymmetric theories fall into supermultiplets, which have both bosonic and fermionic components. There are two types of supermultiplets which appear in renormalizable field theories: chiral and vector supermultiplets.

Chiral supermultiplets are often denoted by the symbol ϕ, which can be (for the purpose of this lecture) regarded as a short-handed notation for the three fields: a complex scalar field A, a Weyl fermion $\frac{1-\gamma_5}{2}\psi = \psi$, and a non-dynamical (auxiliary) complex field F. Lagrangians for chiral supermultiplets consist of two parts, Kähler potential and superpotential. The Kähler potential is nothing but the kinetic terms for the fields, usually written with a short-hand notation $\int d^4\theta \phi^*\phi$, which can be explicitly written down as

$$\mathcal{L} \supset \int d^4\theta \phi_i^* \phi_i = \partial_\mu A_i^* \partial^\mu A_i + \bar{\psi}_i i \gamma^\mu \partial_\mu \psi_i + F_i^* F_i. \qquad (12)$$

Note that the field F does not have derivatives in the Lagrangian and hence is not a propagating field. One can solve for F_i explicitly and eliminate it from the Lagrangian completely.

The superpotential is defined by a holomorphic function $W(\phi)$ of the chiral supermultiplets ϕ_i. A short-hand notation $\int d^2\theta W(\phi)$ gives the following terms in the Lagrangian,

$$\mathcal{L} \supset - \int d^2\theta W(\phi) = -\frac{1}{2} \left. \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \right|_{\phi_i = A_i} \psi^i \psi^j + \left. \frac{\partial W}{\partial \phi_i} \right|_{\phi_i = A_i} F_i. \qquad (13)$$

The first term describes Yukawa couplings between fermionic and bosonic components of the chiral supermultiplets. Using both Eqs. (12) and (13), we can solve for F and find

$$F_i^* = - \left. \frac{\partial W}{\partial \phi_i} \right|_{\phi_i = A_i}. \qquad (14)$$

Substituting it back to the Lagrangian, we eliminate F and instead find a potential term

$$\mathcal{L} \supset -V_F = - \left| \frac{\partial W}{\partial \phi_i} \right|^2_{\phi_i = A_i}. \qquad (15)$$

Vector supermultiplets W_α (α is a spinor index, but never mind), which are supersymmetric generalization of the gauge fields, consist also of three components, a Weyl fermion (gaugino) λ, a vector (gauge) field A_μ, and a non-dynamical (auxiliary) real scalar field D, all in the adjoint representation of the gauge group with the index a. A short-hand notation of their kinetic terms is

$$\mathcal{L} \supset \int d^2\theta W_\alpha^a W^{\alpha a} = -\frac{1}{4} F_{\mu\nu} + \bar{\lambda}^a i \slashed{D} \lambda^a + \frac{1}{2} D^a D^a. \qquad (16)$$

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Note that the field D does not have derivatives in the Lagrangian and hence is not a propagating field. One can solve for D^a explicitly and eliminate it from the Lagrangian completely.

Since the vector supermultiplets contain gauge fields, chiral supermultiplets which transform non-trivially under the gauge group should also couple to the vector multiplets to make the Lagrangian gauge invariant. This requires the modification of the Kähler potential $\int d^4\theta \phi^* \phi$ to $\int d^4\theta \phi^\dagger e^{2gV} \phi$, where V is another short-hand notation of the vector multiplet. Then the kinetic terms in Eq. (12) are then modified to

$$
\mathcal{L} \supset \int d^4\theta \phi_i^\dagger e^{2gV} \phi_i
$$
$$
= D_\mu A_i^\dagger D^\mu A_i + \bar{\psi}_i i\gamma^\mu D_\mu \psi_i + F_i^\dagger F_i - \sqrt{2}g(A^\dagger T^a \lambda^a \psi) - gA^\dagger T^a D^a A.
$$
$$
(17)
$$

Using Eqs. (16,17), one can solve for D^a and eliminate it from the Lagrangian, finding a potential term

$$
\mathcal{L} \supset -V_D = -\frac{g^2}{2}(A^\dagger T^a A)^2 \tag{18}
$$

General supersymmetric Lagrangians are given by Eqs. (17,15,18).[b]

Even though we do not go into formal discussions of supersymmetric field theories, one important theorem must be quoted: the non-renormalization theorem of the superpotential. Under the renormalization of the theories, the superpotential does not receive renormalization at all orders in perturbation theory.[c] We will come back to the virtues of this theorem later on.

Finally, let us study a very simple example of superpotential to gain some intuition. Consider two chiral supermultiplets ϕ_1 and ϕ_2, with a superpotential

$$
W = m\phi_1\phi_2. \tag{19}
$$

Following the above prescription, the fermionic components have the Lagrangian

$$
\mathcal{L} \supset -\frac{1}{2}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi^i\psi^j = -m\psi_1\psi_2, \tag{20}
$$

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[b]We dropped one possible term called Fayet–Illiopoulos D-term possible for vector supermultiplets of Abelian gauge groups. They are often not useful in phenomenological models, but there are exceptions.[11,12]

[c]There are non-perturbative corrections to the superpotential, however. See, *e.g.*, a review.[10]

while the scalar potential term Eq. (15) gives

$$\mathcal{L} \supset - \left| \frac{\partial W}{\partial \phi_i} \right|^2_{\phi_i = A_i} = -m^2 |A_1|^2 - m^2 |A_2|^2. \tag{21}$$

Obviously, the terms Eqs. (20,21) are mass terms for the fermionic (Dirac fermion) and scalar components (two complex scalars) of the chiral supermultiplets, with the same mass m. In general, fermionic and bosonic components in the same supermultiplets are degenerate in supersymmetric theories.

3   Softly Broken Supersymmetry

We've discussed supersymmetric Lagrangians in the previous section, which always give degenerate bosons and fermions. In the real world, we do not see such degenerate particles with the opposite statistics. Therefore supersymmetry must be broken. We will come back later to briefly discuss various mechanisms which break supersymmetry spontaneously in manifestly supersymmetric theories. In the low-energy effective theories, however, we can just add terms to supersymmetric Lagrangians which break supersymmetry explicitly. The important constraint is that such explicit breaking terms should not spoil the motivation discussed earlier, namely to keep the Higgs mass-squared only logarithmically divergent. Such explicit breaking terms of supersymmetry are called "soft" breakings.

The possible soft breaking terms have been classified.[13] In a theory with a renormalizable superpotential

$$W = \frac{1}{2} \mu_{ij} \phi_i \phi_j + \frac{1}{6} \lambda_{ijk} \phi_i \phi_j \phi_k, \tag{22}$$

the possible soft supersymmetry breaking terms have the following forms:

$$m_{ij}^2 A_i^* A_j, \qquad M \lambda \lambda, \qquad \frac{1}{2} b_{ij} \mu_{ij} A_i A_j, \qquad \frac{1}{6} a_{ijk} \lambda_{ijk} A_i A_j A_k. \tag{23}$$

The first one is the masses for scalar components in the chiral supermultiplets, which remove degeneracy between the scalar and spinor components. The next one is the masses for gauginos which remove degeneracy between gauginos and gauge bosons. Finally the last two ones are usually called bilinear and trilinear soft breaking terms with parameters b_{ij} and a_{ijk} with mass dimension one.

In principle, any terms with couplings with positive mass dimensions are candidates of soft supersymmetry breaking terms.[14] Possibilities in theories without gauge singlets are

$$\psi_i \psi_j, \qquad A_i^* A_j A_k, \qquad \psi_i \lambda^a \tag{24}$$

Obviously, the first term is possible only in theories with multiplets with vector-like gauge quantum numbers, and the last term with chiral supermultiplets in the adjoint representation. In the presence of gauge singlet chiral supermultiplets, however, such terms cause power divergences and instabilities, and hence are not soft in general. On the other hand, the Minimal Supersymmetric Standard Model, for instance, does not contain any gauge singlet chiral supermultiplets and hence does admit first two possible terms in Eq. (24). There has been some revived interest in these general soft terms.[15] We will not consider these additional terms in the rest of the discussions. It is also useful to know that terms in Eq. (23) can also induce power divergences in the presence of light gauge singlets and heavy multiplets.[16]

It is instructive to carry out some explicit calculations of Higgs boson self-energy in supersymmetric theories with explicit soft supersymmetry breaking terms. Let us consider the coupling of the Higgs doublet chiral supermultiplet H to left-handed Q and right-handed T chiral supermultiplets,[d] given by the superpotential term

$$W = h_t QTH_u. \tag{25}$$

This superpotential term gives rise to terms in the Lagrangian[e]

$$\mathcal{L} \supset -h_t QTH_u - h_t^2 |\tilde{Q}|^2 |H_u|^2 - h_t^2 |\tilde{T}|^2 |H_u|^2 - m_Q^2 |\tilde{Q}|^2 - m_T^2 |\tilde{T}|^2 - h_t A_t \tilde{Q}\tilde{T}H_u, \tag{26}$$

where m_Q^2, m_T^2, and A_t are soft parameters. Note that the fields Q, T are spinor and $\tilde{Q}$, $\tilde{T}$, H_u are scalar components of the chiral supermultiplets (an unfortunate but common notation in the literature). This explicit Lagrangian allows us to easily work out the one-loop self-energy diagrams for the Higgs doublet H_u, after shifting the field H_u around its vacuum expectation value (this also generates mass terms for the top quark and the scalars which have to be consistently included). The diagram with top quark loop from the first term in Eq. (26) is quadratically divergent (negative). The contractions of $\tilde{Q}$ or $\tilde{T}$ in the next two terms also generate (positive) contributions to the Higgs self-energy. In the absence of soft parameters $m_Q^2 = m_T^2 = 0$, these two contributions precisely cancel with each other, consistent with the non-renormalization theorem which states that no mass terms (superpotential terms) can be generated by renormalizations. However, the explicit breaking

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[d]As will be explained in the next section, the right-handed spinors all need to be charged-conjugated to the left-handed ones in order to be part of the chiral supermultiplets. Therefore the chiral supermultiplet T actually contains the left-handed Weyl spinor $(t_R)^c$. The Higgs multiplet here will be denoted H_u in later sections.

[e]We dropped terms which do not contribute to the Higgs boson self-energy at the one-loop level.

terms m_Q^2, m_T^2 make the cancellation inexact. With a simplifying assumption $m_Q^2 = m_T^2 = \tilde{m}^2$, we find

$$\delta m_H^2 = -\frac{6h_t^2}{(4\pi)^2}\tilde{m}^2 \log \frac{\Lambda^2}{\tilde{m}^2}. \tag{27}$$

Here, Λ is the ultraviolet cutoff of the one-loop diagrams. Therefore, these mass-squared parameters are indeed "soft" in the sense that they do not produce power divergences. Similarly, the diagrams with two $h_t A_t$ couplings with scalar top loop produce only a logarithmic divergent contribution.

4 The Minimal Supersymmetric Standard Model

Encouraged by the discussion in the previous section that the supersymmetry can be explicitly broken while retaining the absence of power divergences, we now try to promote the Standard Model to a supersymmetric theory. The Minimal Supersymmetric Standard Model (MSSM) is a supersymmetric version of the Standard Model with the minimal particle content.

4.1 Particle Content

The first task is to promote all fields in the Standard Model to appropriate supermultiplets. This is obvious for the gauge bosons: they all become vector multiplets. For the quarks and leptons, we normally have left-handed and right-handed fields in the Standard Model. In order to promote them to chiral supermultiplets, however, we need to make all fields left-handed Weyl spinors. This can be done by charge-conjugating all right-handed fields. Therefore, when we refer to supermultiplets of the right-handed down quark, say, we are actually talking about chiral supermultiplets whose left-handed spinor component is the left-handed anti-down quark field. As for the Higgs boson, the field Eq. (1) in the Standard Model can be embedded into a chiral supermultiplet H_u. It can couple to the up-type quarks and generate their masses upon the symmetry breaking. In order to generate down-type quark masses, however, we normally use

$$i\sigma_2 H^* = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix}. \tag{28}$$

Unfortunately, this trick does not work in a supersymmetric fashion because the superpotential W must be a holomorphic function of the chiral supermultiplets and one is not allowed to take a complex conjugation of this sort.

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Table 3. The chiral supermultiplets in the Minimal Supersymmetric Standard Model.. The numbers in the bold face refer to $SU(3)_C$, $SU(2)_L$ representations. The superscripts are hypercharges.

$L_1(\mathbf{1},\mathbf{2})^{-1/2}$	$L_2(\mathbf{1},\mathbf{2})^{-1/2}$	$L_3(\mathbf{1},\mathbf{2})^{-1/2}$
$E_1(\mathbf{1},\mathbf{1})^{+1}$	$E_2(\mathbf{1},\mathbf{1})^{+1}$	$E_3(\mathbf{1},\mathbf{1})^{+1}$
$Q_1(\mathbf{3},\mathbf{2})^{1/6}$	$Q_2(\mathbf{3},\mathbf{2})^{1/6}$	$Q_3(\mathbf{3},\mathbf{2})^{1/6}$
$U_1(\mathbf{3},\mathbf{1})^{-2/3}$	$U_2(\mathbf{3},\mathbf{1})^{-2/3}$	$U_3(\mathbf{3},\mathbf{1})^{-2/3}$
$D_1(\mathbf{3},\mathbf{1})^{+1/3}$	$D_2(\mathbf{3},\mathbf{1})^{+1/3}$	$D_3(\mathbf{3},\mathbf{1})^{+1/3}$
	$H_u(\mathbf{1},\mathbf{2})^{+1/2}$	
	$H_d(\mathbf{1},\mathbf{2})^{-1/2}$	

Therefore, we need to introduce another chiral supermultiplet H_d which has the same gauge quantum numbers of $i\sigma_2 H^*$ above.[f]

In all, the chiral supermultiplets in the Minimal Supersymmetric Standard Model are listed in Table 3.

The particles in the MSSM are referred to as follows.[g] First of all, all quarks, leptons are called just in the same way as in the Standard Model, namely electron, electron-neutrino, muon, muon-neutrino, tau, tau-neutrino, up, down, strange, charm, bottom, top. Their superpartners, which have spin 0, are named with "s" at the beginning, which stand for "scalar." They are denoted by the same symbols as their fermionic counterpart with the tilde. Therefore, the superpartner of the electron is called "selectron," and is written as $\tilde{e}$. All these names are funny, but probably the worst one of all is the "sstrange" ($\tilde{s}$), which I cannot pronounce at all. Superpartners of quarks are "squarks," and those of leptons are "sleptons." Sometimes all of them are called together as "sfermions," which does not make sense at all because they are bosons. The Higgs doublets are denoted by capital H, but as we will see later, their physical degrees of freedom are h^0, H^0, A^0 and $H^{\pm}$. Their superpartners are called "higgsinos," written as $\tilde{H}_u^0$, $\tilde{H}_u^+$, $\tilde{H}_d^-$, $\tilde{H}_d^0$. In general, fermionic superpartners of boson in the Standard Model have "ino" at the end of the name. Spin 1/2 superpartners of the gauge bosons are "gauginos" as mentioned in the previous section, and for each gauge groups:

---

[f] Another reason to need both H_u and H_d chiral supermultiplets is to cancel the gauge anomalies arising from their spinor components.

[g] When I first learned supersymmetry, I didn't believe it at all. Doubling the degrees of freedom looked too much to me, until I came up with my own argument at the beginning of the lecture. The funny names for the particles were yet another reason not to believe in it. It doesn't sound scientific. Once supersymmetry will be discovered, we definitely need better sounding names!

gluino for gluon, wino for W, bino for $U(1)_Y$ gauge boson B. As a result of the electroweak symmetry breaking, all neutral "inos", namely two neutral higgsinos, the neutral wino $\tilde{W}^3$ and the bino $\tilde{B}$ mix with each other to form four Majorana fermions. They are called "neutralinos" $\tilde{\chi}_i^0$ for $i = 1, 2, 3, 4$. Similarly, the charged higgsinos $\tilde{H}_u^+$, $\tilde{H}_d^-$, $\tilde{W}^-$, $\tilde{W}^+$ mix and form two massive Dirac fermions "charginos" $\tilde{\chi}_i^\pm$ for $i = 1, 2$. All particles with tilde do not exist in the non-supersymmetric Standard Model. Once we introduce R-parity in a later section, the particles with tilde have odd R-parity.

4.2 Superpotential

The $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance allows the following terms in the superpotential

$$
\begin{aligned}
W = {}& \lambda_u^{ij} Q_i U_j H_u + \lambda_d^{ij} Q_i D_j H_d + \lambda_e^{ij} L_i E_j H_d + \mu H_u H_d \\
& + \lambda_u^{\prime ijk} U_i D_j D_k + \lambda_d^{\prime ijk} Q_i D_j L_k + \lambda_e^{\prime ijk} L_i E_j L_k + \mu_i' L_i H_u.
\end{aligned}
\tag{29}
$$

The first three terms correspond to the Yukawa couplings in the Standard Model (with exactly the same number of parameters). The subscripts i, j, k are generation indices. The parameter μ has mass dimension one and gives a supersymmetric mass to both fermionic and bosonic components of the chiral supermultiplets H_u and H_d. The terms in the second line of Eq. (29) are in general problematic as they break the baryon (B) or lepton (L) numbers.

   If the superpotential contains both B- and L-violating terms, such as $\lambda_u^{\prime 112} U_1 D_1 D_2$ and $\lambda_d^{\prime 121} Q_1 D_2 L_1$, one can exchange $\tilde{D}_2 = \tilde{s}$ to generate a four-fermion operator

$$
\frac{\lambda_u^{\prime 112} \lambda_d^{\prime 121}}{m_{\tilde{s}}^2} (u_R d_R)(Q_1 L_1),
\tag{30}
$$

where the spinor indices are contracted in each parentheses and the color indices by the epsilon tensor. Such an operator would contribute to the proton decay process $p \to e^+ \pi^0$ at a rate of $\Gamma \sim \lambda'^4 m_p^5 / m_{\tilde{s}}^4$, and hence the partial lifetime of the order of

$$
\tau_p \sim 6 \times 10^{-13} \text{ sec} \left(\frac{m_{\tilde{s}}}{1 \text{ TeV}}\right)^4 \frac{1}{\lambda'^4}.
\tag{31}
$$

Recall that the experimental limit on the proton partial lifetime in this mode is $\tau_p > 1.6 \times 10^{33}$ years.[17] Unless the coupling constants are extremely small, this is clearly a disaster.

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4.3 R-parity

To avoid this problem of too-rapid proton decay, a common assumption is a discrete symmetry called R-parity[18] (or matter parity[19]). The Z_2 discrete charge is given by

$$R_p = (-1)^{2s+3B+L} \tag{32}$$

where s is the spin of the particle. Under R_p, all standard model particles, namely quarks, leptons, gauge bosons, and Higgs bosons, carry even parity, while their superpartners odd due to the $(-1)^{2s}$ factor. Once this discrete symmetry is imposed, all terms in the second line of Eq. (29) will be forbidden, and we do not generate a dangerous operator such as that in Eq. (30). Indeed, B- and L-numbers are now accidental symmetries of the MSSM Lagrangian as a consequence of the supersymmetry, gauge invariance, renormalizability and R-parity conservation.

One immediate consequence of the conserved R-parity is that the lightest particle with odd R-parity, *i.e.*, the Lightest Supersymmetric Particle (LSP), is stable. Another consequence is that one can produce (or annihilate) superparticles only pairwise. These two points have important implications on the collider phenomenology and cosmology. Since the LSP is stable, its cosmological relic is a good (and arguably the best) candidate for the Cold Dark Matter particles (see, *e.g.*, a review[20] on this subject). If so, we do not want it to be electrically charged and/or strongly interacting; otherwise we should have detected them already. Then the LSP should be a superpartner of Z, γ, or neutral Higgs bosons or their linear combination (called neutralino).[h] On the other hand, the superparticles can be produced only in pairs and they decay eventually into the LSP, which escapes detection. This is why the typical signature of supersymmetry at collider experiments is the missing energy/momentum.

The phenomenology of R-parity breaking models has been also studied. If either B-violating or L-violating terms exist in Eq. (29), but not both, they would not induce proton decay.[23] However they can still produce n-$\bar{n}$ oscillation and a plethora of flavor-changing phenomena. We refer to a recent compilation of phenomenological constraints[24] for further details.

---

[h]A sneutrino can in principle be the LSP,[12] but it cannot be the CDM to avoid constraints from the direct detection experiment for the CDM particles.[21] It becomes a viable candidate again if there is a large lepton number violation.[22]

4.4 Soft Supersymmetry Breaking Terms

In addition to the interactions that arise from the superpotential Eq. (29), we should add soft supersymmetry breaking terms to the Lagrangian as we have not seen any of the superpartners of the Standard Model particles. Following the general classifications in Eq. (23), and assuming R-parity conservation, they are given by

$$\mathcal{L}_{soft} = \mathcal{L}_1 + \mathcal{L}_2, \tag{33}$$

$$\mathcal{L}_1 = -m_Q^{2ij}\tilde{Q}_i^*\tilde{Q}_j - m_U^{2ij}\tilde{U}_i^*\tilde{U}_j - m_D^{2ij}\tilde{D}_i^*\tilde{D}_j$$
$$-m_L^{2ij}\tilde{L}_i^*\tilde{L}_j - m_E^{2ij}\tilde{E}_i^*\tilde{E}_j - m_{H_u}^2|H_u|^2 - m_{H_d}^2|H_d|^2, \tag{34}$$

$$\mathcal{L}_2 = -A_u^{ij}\lambda_u^{ij}\tilde{Q}_i\tilde{U}_jH_u - A_d^{ij}\lambda_d^{ij}\tilde{Q}_i\tilde{D}_jH_d - A_l^{ij}\lambda_e^{ij}\tilde{Q}_i\tilde{U}_jH_d + B\mu H_uH_d + c.c. \tag{35}$$

The mass-squared parameters for scalar quarks (squarks) and scalar leptons (sleptons) are all three-by-three hermitian matrices, while the trilinear couplings A^{ij} and the bilinear coupling B of mass dimension one are general complex numbers.[i]

4.5 Higgs Sector

It is of considerable interest to look closely at the Higgs sector of the MSSM. Following the general form of the supersymmetric Lagrangians Eqs. (17,15,18) with the superpotential $W = \mu H_u H_d$ in Eq. (29) as well as the soft parameters in Eq. (34), the potential for the Higgs bosons is given as

$$V = \frac{g'^2}{2}\left(H_u^\dagger\frac{1}{2}H_u + H_d^\dagger\frac{-1}{2}H_d\right)^2 + \frac{g^2}{2}\left(H_u^\dagger\frac{\vec{\tau}}{2}H_u + H_d^\dagger\frac{\vec{\tau}}{2}H_d\right)^2$$
$$+\mu^2(|H_u|^2 + |H_d|^2) + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 - (B\mu H_uH_d + c.c.) \tag{36}$$

It turns out that it is always possible to gauge-rotate the Higgs bosons such that

$$\langle H_u\rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle H_d\rangle \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \tag{37}$$

in the vacuum. Since only electrically neutral components have vacuum expectation values, the vacuum necessarily conserves $U(1)_{\text{QED}}$.[j] Writing the

---

[i]It is unfortunate that the notation A is used both for the scalar components of chiral supermultiplets and the trilinear couplings. Hopefully one can tell them apart from the context.

[j]This is not necessarily true in general two-doublet Higgs Models. Consult a review.[25]

potential (36) down using the expectation values (37), we find

$$V = \frac{g_Z^2}{8}(v_u^2 - v_d^2)^2 + (v_u \ v_d) \begin{pmatrix} \mu^2 + m_{H_u}^2 & -B\mu \\ -B\mu & \mu^2 + m_{H_d}^2 \end{pmatrix} \begin{pmatrix} v_u \\ v_d \end{pmatrix}, \qquad (38)$$

where $g_Z^2 = g^2 + g'^2$. In order for the Higgs bosons to acquire the vacuum expectation values, the determinant of the mass matrix at the origin must be negative,

$$\det \begin{pmatrix} \mu^2 + m_{H_u}^2 & -B\mu \\ -B\mu & \mu^2 + m_{H_d}^2 \end{pmatrix} < 0. \qquad (39)$$

However, there is a danger that the direction $v_u = v_d$, which makes the quartic term in the potential identically vanish, may be unbounded from below. For this not to occur, we need

$$\mu^2 + m_{H_u}^2 + \mu^2 + m_{H_d}^2 > 2\mu B. \qquad (40)$$

In order to reproduce the mass of the Z-boson correctly, we need

$$v_u = \frac{v}{\sqrt{2}} \sin\beta, \qquad v_d = \frac{v}{\sqrt{2}} \cos\beta, \qquad v = 250 \text{ GeV}. \qquad (41)$$

The vacuum minimization conditions are given by $\partial V/\partial v_u = \partial V/\partial v_d = 0$ from the potential Eq. (38). Using Eq. (41), we obtain

$$\mu^2 = -\frac{m_Z^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}, \qquad (42)$$

and

$$B\mu = (2\mu^2 + m_{H_u}^2 + m_{H_d}^2) \sin\beta \cos\beta. \qquad (43)$$

Because there are two Higgs doublets, each of which with four real scalar fields, the number of degrees of freedom is eight before the symmetry breaking. However three of them are eaten by W^+, W^- and Z bosons, and we are left with five physics scalar particles. There are two CP-even scalars h^0, H^0, one CP-odd scalar A^0, and two charged scalars H^+ and H^-. Their masses can be worked out from the potential (38):

$$m_A^2 = 2\mu^2 + m_{H_u}^2 + m_{H_d}^2, \qquad m_{H^\pm}^2 = m_W^2 + m_A^2, \qquad (44)$$

and

$$m_{h^0}^2, m_{H^0}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta}\right). \qquad (45)$$

A very interesting consequence of the formula Eq. (45) is that the lighter CP-even Higgs mass $m_{h^0}^2$ is maximized when $\cos^2 2\beta = 1$: $m_{h^0}^2 = (m_A^2 + m_Z^2 -$

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$|m_A^2 - m_Z^2|)/2$. When $m_A < m_Z$, we obtain $m_{h^0}^2 = m_A^2 < m_Z^2$, while when $m_A > m_Z$, $m_{h^0}^2 = m_Z^2$. Therefore in any case we find

$$m_{h^0} \leq m_Z. \tag{46}$$

This is an important prediction in the MSSM. The reason why the masses of the Higgs boson are related to the gauge boson masses is that the Higgs quartic couplings in Eq. (36) are all determined by the gauge couplings because they originate from the elimination of the auxiliary D-fields in Eq. (17).

Unfortunately, the prediction Eq. (46) is modified at the one-loop level,[26] approximately as

$$\Delta(m_{h^0}^2) = \frac{N_c}{4\pi^2} h_t^4 v^2 \sin^4 \beta \log \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right). \tag{47}$$

With the scalar top mass of up to 1 TeV, the lightest Higgs mass is pushed up to about 130 GeV. (See also the latest analysis including resummed two-loop contribution.[27])

The parameter space of the MSSM Higgs sector can be described by two parameters. This is because the potential Eq. (38) has three independent parameters, $\mu^2 + m_{H_u}^2$, $\mu^2 + m_{H_d}^2$, and $B\mu$, while one combination is fixed by the Z-mass Eq. (39). It is customary to pick either $(m_A, \tan \beta)$, or $(m_{h^0}, \tan \beta)$ to present experimental constraints. The current experimental constraint on this parameter space is shown in Fig. 1.[k]

The range of the Higgs mass predicted in the MSSM is not necessarily an easy range for the LHC experiments, but three-years' running at the high luminosity is supposed to cover the entire MSSM parameter space, by employing many different production/decay modes as seen in Fig. 2.

4.6   Neutralinos and Charginos

Once the electroweak symmetry is broken, and since supersymmetry is already explicitly broken in the MSSM, there is no quantum number which can distinguish two neutral higgsino states $\tilde{H}_u^0$, $\tilde{H}_d^0$, and two neutral gaugino states $\tilde{W}^3$ (neutral wino) and $\tilde{B}$ (bino). They have four-by-four Majorana mass matrix

$$\mathcal{L} \supset -\frac{1}{2} \times$$

---

[k]The large $\tan \beta$ region may appear completely excluded in the plot, but this is somewhat misleading; it is due to the parameterization $(m_{h^0}, \tan \beta)$ which squeezes the m_{h^0} region close to the theoretical upper bound to a very thin one. In the $(m_A, \tan \beta)$ parameterization, one can see the allowed region much more clearer.
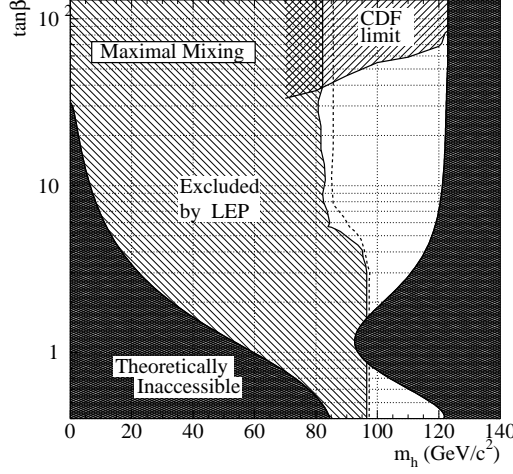
Figure 1. Regions in the $(m_{h^0}, \tan\beta)$ plane excluded by the MSSM Higgs boson searches at LEP in data up to 189 GeV, and at CDF in run I data. The regions not allowed by the MSSM for a top mass of 175 GeV, a SUSY scale of 1 TeV and maximal mixing in the stop sector are also indicated. The dotted curve is the LEP expected limit.[28]
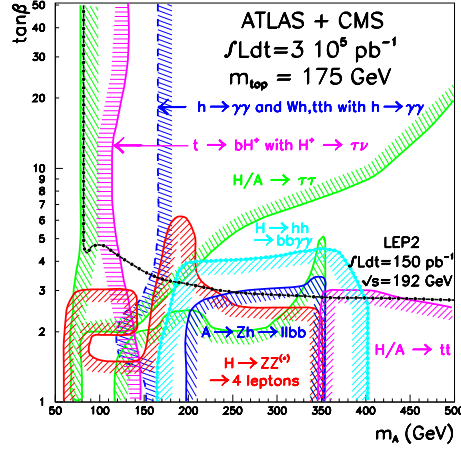

Figure 2. Expected coverage of the MSSM Higgs sector parameter space by the LHC experiments, after three years of high-luminosity running.

$$
(\tilde{B} \ \tilde{W}^3 \ \tilde{H}_d^0 \ \tilde{H}_u^0)
\begin{pmatrix}
M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\
0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\
-m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\
m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0
\end{pmatrix}
\begin{pmatrix}
\tilde{B} \\
\tilde{W}^3 \\
\tilde{H}_d^0 \\
\tilde{H}_u^0
\end{pmatrix}.
$$

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$$\tag{48}$$

Here, $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, $s_\beta = \sin\beta$, and $c_\beta = \cos\beta$. Once M_1, M_2, μ exceed m_Z, which is preferred given the current experimental limits, one can regard components proportional to m_Z as small perturbations. Then the neutralinos are close to their weak eigenstates, bino, wino, and higgsinos. But the higgsinos in this limit are mixed to form symmetric and anti-symmetric linear combinations $\tilde{H}_S^0 = (\tilde{H}_d^0 + \tilde{H}_u^0)/\sqrt{2}$ and $\tilde{H}_A^0 = (\tilde{H}_d^0 - \tilde{H}_u^0)/\sqrt{2}$.

Similarly two positively charged inos: $\tilde{H}_u^+$ and $\tilde{W}^+$, and two negatively charged inos: $\tilde{H}_d^-$ and $\tilde{W}^-$ mix. The mass matrix is given by

$$\mathcal{L} \supset -(\tilde{W}^- \ \tilde{H}_d^-) \begin{pmatrix} M_2 & \sqrt{2}m_W s_\beta \\ \sqrt{2}m_W c_\beta & \mu \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} + c.c. \tag{49}$$

Again once $M_2, \mu \gtrsim m_W$, the chargino states are close to the weak eigenstates winos and higgsinos.

4.7 Squarks, Sleptons

The mass terms of squarks and sleptons are also modified after the electroweak symmetry breaking. There are four different contributions. One is the supersymmetric piece coming from the $|\partial W/\partial\phi_i|^2$ terms in Eq. (15) with $\phi_i = Q, U, D, L, E$. These terms add m_f^2 where m_f is the mass of the quarks and leptons from their Yukawa couplings to the Higgs boson. Next one is combing from the $|\partial W/\partial\phi_i|^2$ terms in Eq. (15) with $\phi_i = H_u$ or H_d in the superpotential Eq. (29). Because of the μ term,

$$\frac{\partial W}{\partial H_u^0} = -\mu H_d^0 + \lambda_u^{ij}\tilde{Q}_i\tilde{U}_j, \tag{50}$$

$$\frac{\partial W}{\partial H_d^0} = -\mu H_d^0 + \lambda_d^{ij}\tilde{Q}_i\tilde{D}_j + \lambda_e^{ij}\tilde{L}_i\tilde{E}_j. \tag{51}$$

Taking the absolute square of these two expressions pick the cross terms together with $\langle H_d^0\rangle = v\cos\beta/\sqrt{2}$, $\langle H_u^0\rangle = v\sin\beta/\sqrt{2}$ and we obtain mixing between $\tilde{Q}$ and $\tilde{U}$, $\tilde{Q}$ and $\tilde{D}$, and $\tilde{L}$ and $\tilde{E}$. Similarly, the vacuum expectation values of the Higgs bosons in the trilinear couplings Eq. (35) also generate similar mixing terms. Finally, the D-term potential after eliminating the auxiliary field D Eq. (18) also give contributions to the scalar masses $m_Z^2(I_3 - Q\sin^2\theta_W)\cos 2\beta$. Therefore, the mass matrix of stop, for instance, is given as

$$\mathcal{L} \supset -(\tilde{t}_L^* \ \tilde{t}_R^*)$$

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$$\begin{pmatrix} m_{Q_3}^2 + m_t^2 + m_Z^2(\frac{1}{2} - \frac{2}{3}s_W^2)c_{2\beta} & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & m_{U_3}^2 + m_t^2 + m_Z^2(-\frac{2}{3}s_W^2)c_{2\beta} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix},$$

(52)

with $c_{2\beta} = \cos 2\beta$. Here, $\tilde{t}_L$ is the up component of $\tilde{Q}_3$, and $\tilde{t}_R = \tilde{T}^*$. For first and second generation particles, the off-diagonal terms are negligible for most purposes. They may, however, be important when their loops in flavor-changing processes are considered.

4.8 What We Gained in the MSSM

It is useful to review here what we have gained in the MSSM over what we had in the Standard Model. The main advantage of the MSSM is of course what motivated the supersymmetry to begin with: the absence of the quadratic divergences as seen in Eq. (27). This fact allows us to apply the MSSM down to distance scales much shorter than the electroweak scale, and hence we can at least hope that many of the puzzles discussed at the beginning of the lecture to be solved by physics at the short distance scales.

There are a few amusing and welcome by-products of supersymmetry beyond this very motivation. First of all, the Higgs doublet in the Standard Model appears so unnatural partly because it is the only scalar field introduced just for the sake of the electroweak symmetry breaking. In the MSSM, however, there are so many scalar fields: 15 complex scalar fields for each generation and two in each Higgs doublet. Therefore, the Higgs bosons are just "one of them." Then the question about the electroweak symmetry breaking is addressed in a completely different fashion: why is it only the Higgs bosons that condense? In fact, one can even partially answer this question in the renormalization group analysis in the next sections where "typically" (we will explain what we mean by this) it is only the Higgs bosons which acquire negative mass squared (39) while the masses-squared of all the other scalars "naturally" remain positive. Finally, the absolute upper bound on the lightest CP-even Higgs boson is falsifiable by experiments.

However, life is not as good as we wish. We will see that there are very stringent low-energy constraints on the MSSM in the next section.

5 Low-Energy Constraints

Despite the fact that we are interested in superparticles in the 100–1000 GeV range, which we are just starting to explore in collider searches, there are many amazingly stringent low-energy constraints on superparticles.
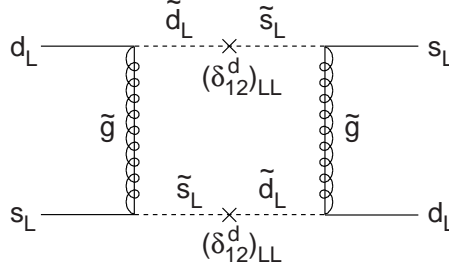
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Figure 3. A Feynman diagram which gives rise to Δm_K and ε_K.

One of the most stringent constraints comes from the K^0–$\bar{K}^0$ mixing parameters Δm_K and ε_K. The main reason for the stringent constraints is that the scalar masses-squared in the MSSM Lagrangian Eq. (34) can violate flavor, *i.e.*, the scalar masses-squared matrices are not necessarily diagonal in the basis where the corresponding quark mass matrices are diagonal.

To simplify the discussion, let us concentrate only on the first and the second generations (ignore the third). We also go to the basis where the down-type Yukawa matrix λ_d^{ij} is diagonal, such that

$$\lambda_d^{ij} v_d = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}. \tag{53}$$

Therefore the states $K^0 = (d\bar{s})$, $\bar{K}^0 = (s\bar{d})$ are well-defined in this basis. In the same basis, however, the squark masses-squared can have off-diagonal elements in general,

$$m_Q^{2ij} = \begin{pmatrix} m_{\tilde{d}_L}^2 & m_{Q,12}^2 \\ m_{Q,12}^{2*} & m_{\tilde{s}_L}^2 \end{pmatrix}, \qquad m_D^{2ij} = \begin{pmatrix} m_{\tilde{d}_R}^2 & m_{D,12}^2 \\ m_{D,12}^{2*} & m_{\tilde{s}_R}^2 \end{pmatrix}. \tag{54}$$

Since their off-diagonal elements will be required to be small (as we will see later), it is convenient to treat them as small perturbation. We insert the off-diagonal elements as two-point Feynman vertices which change the squark flavor $\tilde{d}_{L,R} \leftrightarrow \tilde{s}_{L,R}$ in the diagrams. To simplify the discussion further, we assume that all squarks and gluino the are comparable in their masses $\tilde{m}$. Then the relevant quantities are given in terms of the ratio $(\delta_{12}^d)_{LL} \equiv m_{Q,12}^2/\tilde{m}^2$ (and similarly $(\delta_{12}^d)_{RR} = m_{D,12}^2/\tilde{m}^2$), as depicted in Fig. 3. The operator from this Feynman diagram is estimated approximately as

$$0.005\alpha_s^2 \frac{(\delta_{12}^d)_{LL}^2}{\tilde{m}^2}(\bar{d}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu s_L). \tag{55}$$

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This operator is further sandwiched between K^0 and $\bar{K}^0$ states, and we find

$$\Delta m_K^2 \sim 0.005 f_K^2 m_K^2 \alpha_s^2 (\delta_{12}^d)_{LL}^2 \frac{1}{\tilde{m}^2}$$

$$= 1.2 \times 10^{-12} \text{ GeV}^2 \left(\frac{f_K}{160 \text{ MeV}}\right)^2 \left(\frac{\alpha_s}{0.1}\right)^2 (\delta_{12}^d)_{LL}^2 < 3.5 \times 10^{-15} \text{ GeV}^2,$$

$$(56)$$

where the last inequality is the phenomenological constraint in the absence of accidental cancellations. This requires

$$(\delta_{12}^d)_{LL} \lesssim 0.05 \left(\frac{\tilde{m}}{500 \text{ GeV}}\right) \tag{57}$$

and hence the off-diagonal element $m_{Q,12}^2$ must be small. It turns out that the product $(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}$ is more stringently constrained, especially its imaginary part from ε_K. Much more careful and detailed analysis than the above order-of-magnitude estimate gives[29]

$$\text{Re}\left[(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}\right] < (0.016)^2, \qquad \text{Im}\left[(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}\right] < (0.0022)^2. \tag{58}$$

There are many other low-energy observables, such as electron and neutron electric dipole moments (EDM), $\mu \to e\gamma$, which place important constraints on the supersymmetry parameters.[30]

There are various ways to avoid such low-energy constraints on supersymmetry. The first one is called "universality" of soft parameters.[31] It is simply assumed that the scalar masses-squared matrices are proportional to identity matrices, *i.e.*, $m_Q^2, m_U^2, m_D^2 \propto \mathbf{1}$. Then no matter what rotation is made in order to go to the basis where the quark masses are diagonal, the identity matrices stay the same, and hence the off-diagonal elements are never produced. There has been many proposals to generate universal scalar masses either by the mediation mechanism of the supersymmetry breaking such as the gauge mediated (see reviews[32]), anomaly mediated[33], or gaugino mediated[34] supersymmetry breaking, or by non-Abelian flavor symmetries.[35] The second possibility is called "alignment," where certain flavor symmetries should be responsible for "aligning" the quark and squark mass matrices such that the squark masses are almost diagonal in the same basis where the down-quark masses are diagonal.[36] Because of the CKM matrix it is impossible to do this both for down-quark and up-quark masses. Since the phenomenological constraints in the up-quark sector are much weaker than in the down-quark sector, this choice would alleviate many of the low-energy constraints (except for flavor-diagonal CP-violation such as EDMs). Finally there is a possibility

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called "decoupling," which assumes first- and second-generation superpart-
ners much heavier than TeV while keeping the third-generation superpart-
ners as well as gauginos in the 100 GeV range to keep the Higgs self-energy
small enough.[37] Even though this idea suffers from a fine-tuning problem in
general,[38] many models had been constructed to achieve such a split mass
spectrum recently.[39]

In short, the low-energy constraints are indeed very stringent, but there
are many ideas to avoid such constraints naturally within certain model frame-
works. Especially given the fact that we still do not know any of the superpar-
ticle masses experimentally, one cannot make the discussions more clear-cut at
this stage. On the other hand, important low-energy effects of supersymmetry
are still being discovered in the literature, such as muon $g - 2$,[40] and direct
CP-violation.[41] They may be even more possible low-energy manifestations
of supersymmetry which have been missed so far.

6   Renormalization Group Analyses

Once supersymmetry protects the Higgs self-energy against corrections from
the short distance scales, or equivalently, the high energy cutoff scales, it
becomes important to connect physics at the electroweak scale where we
can do measurements to the fundamental parameters defined at high energy
scales. This can be done by studying the renormalization-group evolution
of parameters. It also becomes a natural expectation that the supersym-
metry breaking itself originates at some high energy scale. If this is the
case, the soft supersymmetry breaking parameters should also be studied
using the renormalization-group equations. We study the renormalization-
group evolution of various parameters in the softly-broken supersymmetric
Lagrangian at the one-loop level.[l] If supersymmetry indeed turns out to be
the choice of nature, the renormalization-group analysis will be crucial in
probing physics at high energy scales using the observables at the TeV-scale
collider experiments.[44]

6.1   Gauge Coupling Constants

The first parameters to be studied are naturally the coupling constants in
the Standard Model. The running of the gauge couplings constants are de-
scribed in term of the beta functions, and their one-loop solutions in non-

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[l]Recently, there have been developments in obtaining and understanding all-order beta
functions for gauge coupling constants[42] and soft parameters.[43]

supersymmetric theories are given by

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\mu')} + \frac{b_0}{8\pi^2} \log \frac{\mu}{\mu'}, \tag{59}$$

with

$$b_0 = \frac{11}{3}C_2(G) - \frac{2}{3}S_f - \frac{1}{3}S_b. \tag{60}$$

This formula is for Weyl fermions f and complex scalars b. The group theory factors are defined by

$$\delta^{ad}C_2(G) = f^{abc}f^{dbc} \tag{61}$$
$$\delta^{ab}S_{f,b} = \text{Tr}T^aT^b \tag{62}$$

and $C_2(G) = N_c$ for $\text{SU}(N_c)$ groups and $S_{f,b} = 1/2$ for their fundamental representations.

In supersymmetric theories, there is always the gaugino multiplet in the adjoint representation of the gauge group. They contribute to Eq. (60) with $S_f = C_2(G)$, and therefore the total contribution of the vector supermultiplet is $3C_2(G)$. On the other hand, the chiral supermultiplets have a Weyl spinor and a complex scalar, and the last two terms in Eq. (60) are always added together to $S_f = S_b$. Therefore, the beta function coefficients simplify to

$$b_0 = 3C_2(G) - S_f. \tag{63}$$

Given the beta functions, it is easy to work out how the gauge coupling constants measured accurately at LEP/SLC evolve to higher energies.

One interesting possibility is that the gauge groups in the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ may be embedded into a simple group, such as $SU(5)$ or $SO(10)$, at some high energy scale, called "grand unification." The gauge coupling constants at $\mu \sim m_Z$ are approximately $\alpha^{-1} = 129$, $\sin^2 \theta_W \simeq 0.232$, and $\alpha_s^{-1} = 0.119$. In the $SU(5)$ normalization, the $U(1)$ coupling constant is given by $\alpha_1 = \frac{5}{3}\alpha' = \frac{5}{3}\alpha/\cos^2\theta_W$. It turns out that the gauge coupling constants become equal at $\mu \simeq 2\times10^{16}$ GeV given the MSSM particle content (Fig. 4). On the other hand, the three gauge coupling constants miss each other quite badly with the non-supersymmetric Standard Model particle content. This observation suggests the possibility of supersymmetric grand unification.

6.2 Yukawa Coupling Constants

Since first- and second-generation Yukawa couplings are so small, let us ignore them and concentrate on the third-generation ones. Their renormalization-
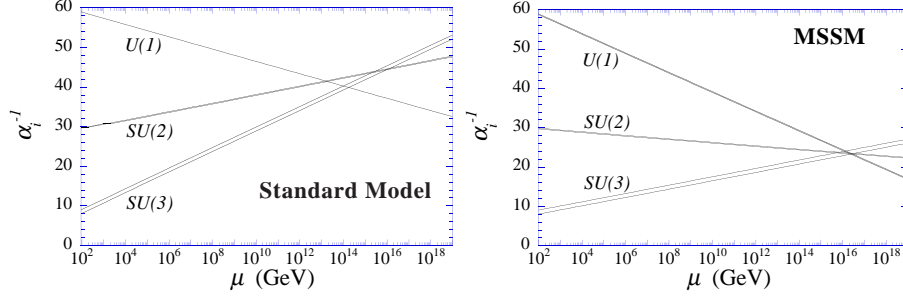
Figure 4. Running of gauge coupling constants in the Standard Model and in the MSSM.

group equations are given as

$$\mu\frac{dh_t}{d\mu} = \frac{h_t}{16\pi^2}\left[6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right], \qquad (64)$$

$$\mu\frac{dh_b}{d\mu} = \frac{h_b}{16\pi^2}\left[6h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right], \qquad (65)$$

$$\mu\frac{dh_\tau}{d\mu} = \frac{h_\tau}{16\pi^2}\left[4h_\tau^2 + 3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right]. \qquad (66)$$

The important aspect of these equations is that the gauge coupling constants push down the Yukawa coupling constants at higher energies, while the Yukawa couplings push them up. This interplay, together with a large top Yukawa coupling, allows the possibility that the Yukawa couplings may also unify at the same energy scale where the gauge coupling constants appear to unify (Fig. 5). It turned out that the actual situation is much more relaxed than what this plot suggests. This is because there is a significant correction to m_b at $\tan\beta \gtrsim 10$ when the superparticles are integrated out [45].

6.3 Soft Parameters

Since we do not know any of the soft parameters at this point, we cannot use the renormalization-group equations to probe physics at high energy scales. On the other hand, we can use the renormalization-group equations from boundary conditions at high energy scales suggested by models to obtain useful information on the "typical" superparticle mass spectrum.
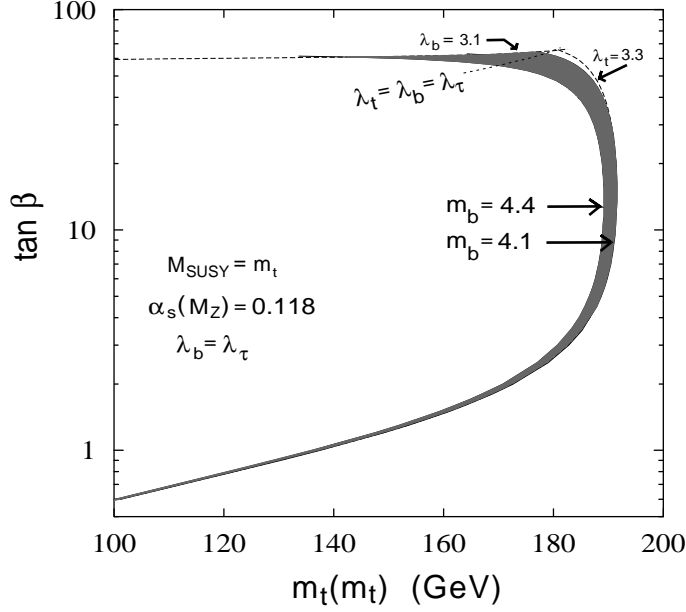
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Figure 5. The regions on $(m_t, \tan\beta)$ plane where $h_b = h_\tau$ at the GUT-scale.[46]

First of all, the gaugino mass parameters have very simple behavior that

$$\mu\frac{d}{d\mu}\frac{M_i}{g_i^2} = 0. \tag{67}$$

Therefore, the ratios M_i/g_i^2 are constants at all energies. If the grand unification is true, both the gauge coupling constants and the gaugino mass parameters must unify at the GUT-scale and hence the ratios are all the same at the GUT-scale. Since the ratios do not run, the ratios are all the same at any energy scales, and hence the low-energy gaugino mass ratios are predicted to be

$$M_1 : M_2 : M_3 = g_1^2 : g_2^2 : g_3^2 \sim 1 : 2 : 7 \tag{68}$$

at the TeV scale. We see the tendency that the colored particle (gluino in this case) is much heavier than uncolored particle (wino and bino in this case). This turns out to be a relatively model-independent conclusion.

The running of scalar masses is given by simple equations when all Yukawa

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couplings other than that of the top quark are neglected. We find

$$16\pi^2\mu\frac{d}{d\mu}m_{H_u}^2 = 3X_t - 6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2, \tag{69}$$

$$16\pi^2\mu\frac{d}{d\mu}m_{H_d}^2 = -6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2, \tag{70}$$

$$16\pi^2\mu\frac{d}{d\mu}m_{Q_3}^2 = X_t - \frac{32}{3}g_3^2M_3^2 - 6g_2^2M_2^2 - \frac{2}{15}g_1^2M_1^2, \tag{71}$$

$$16\pi^2\mu\frac{d}{d\mu}m_{U_3}^2 = 2X_t - \frac{32}{3}g_3^2M_3^2 - \frac{32}{15}g_1^2M_1^2. \tag{72}$$

Here, $X_t = 2h_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{U_3}^2)$ and the trilinear couplings are also neglected. Even within this simplified assumptions, one learns interesting lessons. First of all, the gauge interactions push the scalar masses up at lower energies due to the gaugino mass squared contributions. Colored particles are pushed up even more than uncolored ones, and the right-handed sleptons would be the least pushed up. On the other hand, Yukawa couplings push the scalar masses down at lower energies. The coefficients of X_t in the Eqs. (69, 71, 72) are simply the multiplicity factors which correspond to 3 of $SU(3)_C$, 2 of $SU(2)_Y$ and 1 of $U(1)_Y$. It is extremely amusing that the $m_{H_u}^2$ is pushed down the most because of the factor of three as well as is pushed up the least because of the absence of the gluino mass contribution. Therefore, the fact that the Higgs mass squared is negative at the electroweak scale may well be just a simple consequence of the renormalization-group equations! Since the Higgs boson is just "one of them" in the MSSM, the renormalization-group equations provide a very compelling reason why it is only the Higgs boson whose mass-squared goes negative and condenses. One can view this as an explanation for the electroweak symmetry breaking.

6.4   Minimal Supergravity

Of course, nothing quantitative can be said unless one makes some specific assumptions for the boundary conditions of the renormalization-group equations. One common choice called "Minimal Supergravity" is the following set of assumptions:

$$m_Q^{2ij} = m_U^{2ij} = m_D^{2ij} = m_L^{2ij} = m_E^{2ij} = m_0^2\delta^{ij},$$
$$m_{H_u}^2 = m_{H_d}^2 = m_0^2,$$
$$A_u^{ij} = A_d^{ij} = A_l^{ij} = A_0$$
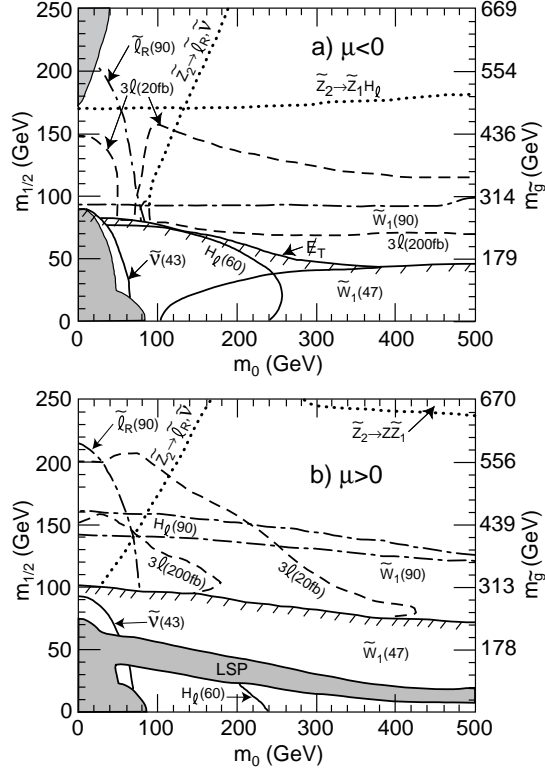$$M_1 = M_2 = M_3 = M_{1/2}$$

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at the GUT-scale. The parameter m_0 is called the universal scalar mass, A_0 the universal trilinear coupling, and $M_{1/2}$ the universal gaugino mass. Once this assumption is made, there are only five parameters at the GUT-scale, $(m_0, M_{1/2}, A_0, B, \mu)$. This assumption also avoids most of the low-energy constraints easily because the scalar mass-squared matrices are proportional to the identity matrices and hence there is no flavor violation. Of course this is probably an oversimplification of the parameter space, but it still provides useful starting point in discussing phenomenology. Especially most of the search limits from collider experiments have been reported using this assumption. In general, this choice of the boundary conditions, which actually have not much to do with supergravity itself, lead to acceptable and interesting phenomenology including the collider signatures, low-energy constraints as well as cosmology.

7 Collider Phenomenology

We do not go into much details of the collider phenomenology of supersymmetry in this lecture notes and we refer to reviews.[47] Here, we give only a very brief summary of collider phenomenology. Supersymmetry is an ideal target for current and new future collider searches. As long as they are within the mass scale expected by the argument given at the beginning of the lecture, we expect supersymmetric particles to be discovered at LEP-II (even though the phase space left is quite limited by now), Tevatron Run-II, or the LHC.

The next two figures Figs. 6, 7 show the discovery reach of supersymmetry at LEP-II, Tevatron Run II, LHC. It is fair to say that the mass range of superparticles relevant to solve the problem of fine cancellation in the Higgs boson self-energy described at the beginning of the lecture is covered by these experiments.

A future e^+e^- linear collider would play a fantastic role in proving that new particles are indeed superpartners of the known Standard Model particles and in determining their parameters.[50] Once such studies will be done, we will exploit renormalization-group analyses trying to connect physics at TeV scale to yet-more-fundamental physics at higher energy scales. Example of such possible studies are shown in Fig. 9. The measurements of gaugino masses were simulated. At the LHC, the measurements are basically on the gluino mass and the LSP mass which is assumed to be the bino state, and their mass difference can be measured quite well. By assuming a value of the LSP mass, one can extract the gluino mass. At the e^+e^- linear colliders, one can even disentangle the mixing in neutralino and chargino states employing expected high beam polarizations and determine M_1 and M_2 in a model-independent

250 — 669
$\tilde{\ell}_R(90)$
a) $\mu<0$
200 — 554
$3\ell(20fb)$ $\tilde{Z}_2\to\tilde{Z}_1H_\ell$
$m_{1/2}$ (GeV) — $m_{\tilde{g}}$ (GeV)
150 — 436
100 — 314
$\tilde{W}_1(90)$
$3\ell(200fb)$
50 — 179
$\tilde{\nu}(43)$ $H_\ell(60)$ $\not{E}_T$
$\tilde{W}_1(47)$
0
0 100 200 300 400 500
m_0 (GeV)

250 — 670
$\tilde{\ell}_R(90)$ $\tilde{Z}_2\to z\tilde{Z}_1$
b) $\mu>0$
200 — 556
$m_{1/2}$ (GeV) — $m_{\tilde{g}}$ (GeV)
$H_\ell(90)$
150 — 439
$3\ell(200fb)$ $3\ell(20fb)$ $\tilde{W}_1(90)$
100 — 313
$\tilde{\nu}(43)$
50 — 178
$\tilde{W}_1(47)$
LSP
$H_\ell(60)$
0
0 100 200 300 400 500
m_0 (GeV)

Baer 01 16GM

Figure 6. Regions in the m_0 $vs.m_{1/2}$ plane explorable by Tevatron and LEP II experiments.[47]

matter. Combination of both types of experiments determine all three gaugino masses, which would provide a non-trivial test of the grand unification.

8   Mediation Mechanisms of Supersymmetry Breaking

One of the most important questions in the supersymmetry phenomenology is how supersymmetry is broken and how the particles in the MSSM learn the effect of supersymmetry breaking. The first one is the issue of dynamical supersymmetry breaking, and the second one is the issue of the "mediation"
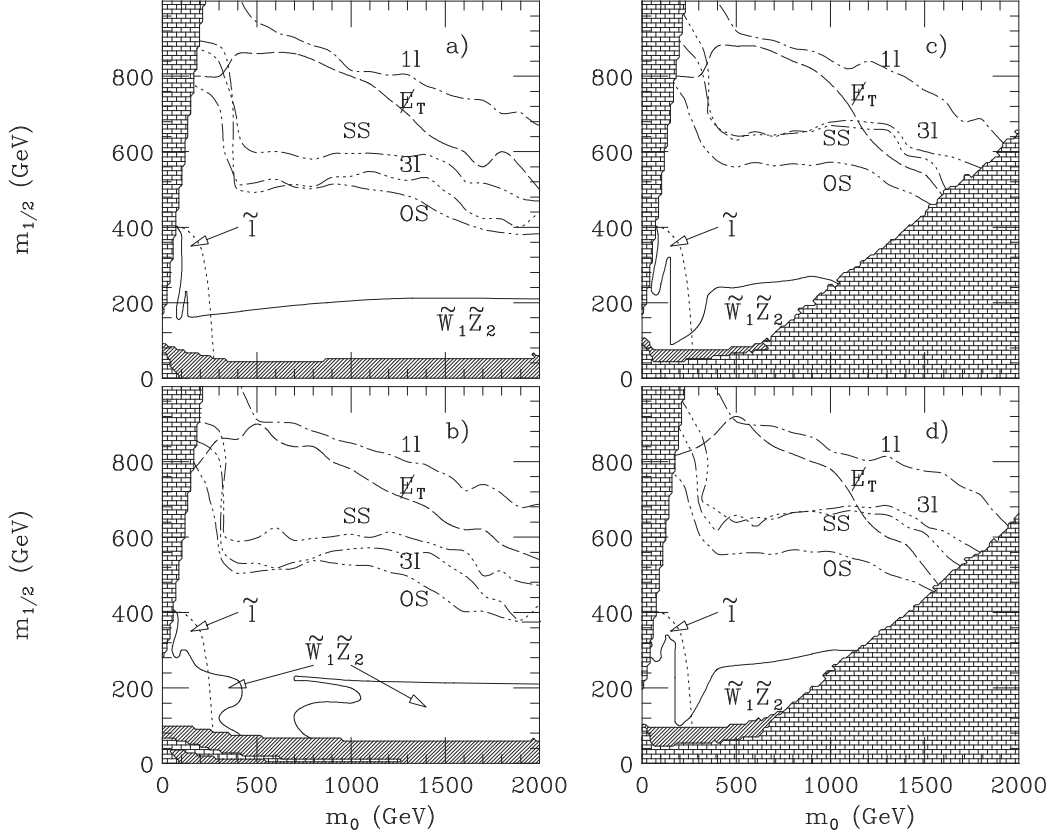
Figure 7. Regions in the m_0 $vs.m_{1/2}$ plane explorable by LHC experiments with 10 fb^{-1} of integrated luminosity.[48] Different curves correspond to different search modes: 1l (single lepton), $\not{E}_T$ (missing transverse energy), SS (same sign dilepton), 3l (trilepton), OS (opposite sign dilepton), $\tilde{l}$ (slepton), $\tilde{W}_1\tilde{Z}_2$ (charged wino, neutral wino associated production).

mechanism.

The problem of the supersymmetry breaking itself has gone through a dramatic progress in the last few years thanks to works on the dynamics of supersymmetric gauge theories by Seiberg.[10] The original idea by Witten[5] was that the dynamical supersymmetry breaking is ideal to explain the hierarchy. Because of the non-renormalization theorem, if supersymmetry is unbroken at the tree-level, it remains unbroken at all orders in perturbation theory.
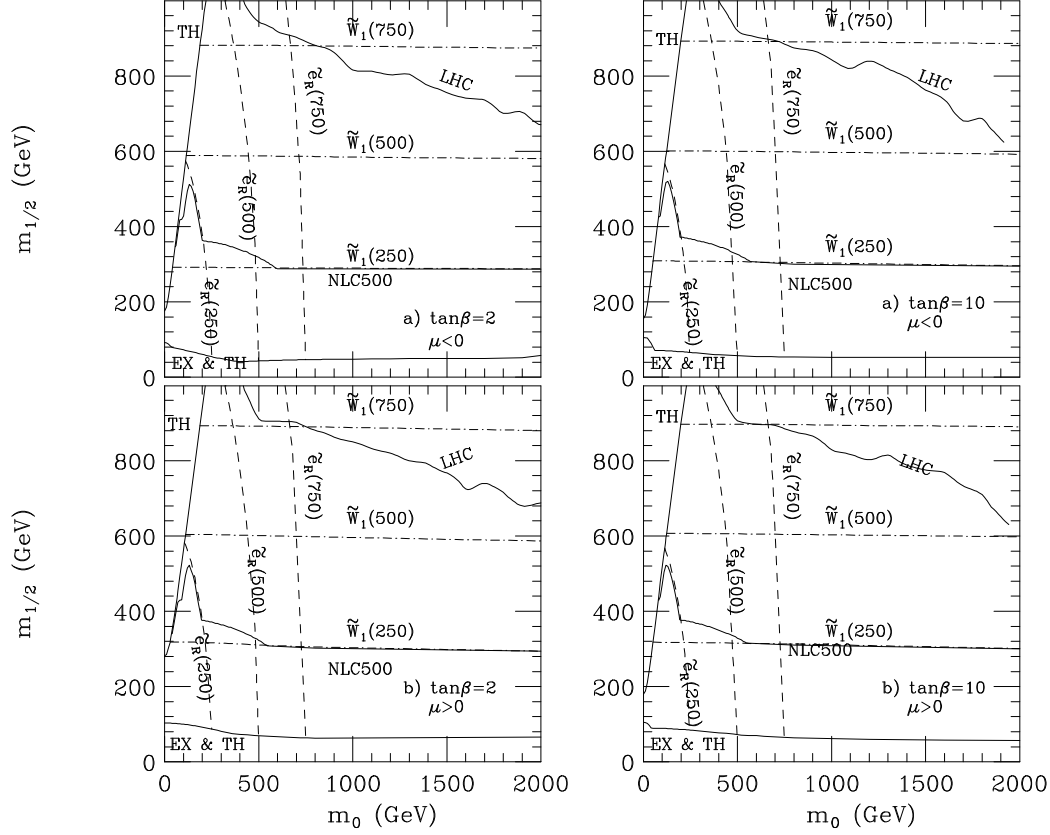
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Figure 8. Regions in the m_0 $vs.m_{1/2}$ plane explorable by e^+e^- linear collider experiments with 20 fb^{-1} of integrated luminosity.

However, they may be non-perturbative effects suppressed by $e^{-8\pi^2/g^2}$ that could break supersymmetry. Then the energy scale of the supersymmetry breaking can be naturally suppressed exponentially compared to the energy scale of the fundamental theory (string?). Even though this idea attracted a lot of interest,[m] the model building was hindered by the lack of understanding in dynamics of supersymmetric gauge theories. Only relative few models were convincingly shown to break supersymmetry dynamically, such as the

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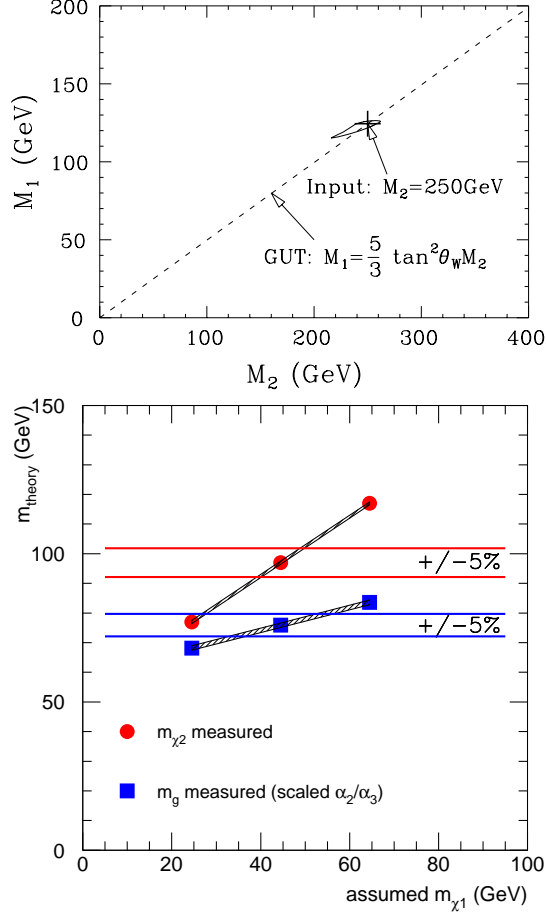[m]I didn't live through this era, so this is just a guess.

Figure 9. Experimental tests of gaugino mass unification at a future e^+e^- collider[50] and the LHC.[51]

$SU(5)$ model with two pairs[52] of $\mathbf{5^*} + \mathbf{10}$ and the 3-2 model.[53] After Seiberg's works, however, there has been an explosion in the number of models which break supersymmetry dynamically (see a review[54] and references therein). For instance, some of the models which were claimed to break supersymmetry dynamically, such as $SU(5)$ with one pair[55] of $\mathbf{5^*} + \mathbf{10}$ or $SO(10)$ with one spinor[56] $\mathbf{16}$, are actually strongly coupled and could not be analyzed reliably (called "non-calculable"), but new techniques allowed us to analyze these

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strongly coupled models reliably.[57] Unexpected vector-like models were also found[58] which proved to be useful for model building.

There has also been an explosion in the number of mediation mechanisms proposed in the literature. The oldest mechanism is that in supergravity theories where interactions suppressed by the Planck scale are responsible for communicating the effects of supersymmetry breaking to the particles in the MSSM. For instance, see a review.[59] Even though the gravity itself may not be the only effect for the mediation but there could be many operators suppressed by the Planck-scale responsible for the mediation, this mechanism was sometimes called "gravity-mediation." The good thing about this mechanism is that this is almost always there. However we basically do not have any control over the Planck-scale physics and the resulting scalar masses-squared are in general highly non-universal. In this situation, the best idea is probably to constrain the scalar masses-squared matrix proportional to the identity matrix by non-Abelian flavor symmetries.[35] Models were constructed where the breaking patterns of the flavor symmetry naturally explain the hierarchical quark and lepton mass matrices, while protecting the squark masses-squared matrices from deviating too far from the identity matrices.

A beautiful idea to guarantee the universal scalar masses is to use the MSSM gauge interactions for the mediation. Then the supersymmetry breaking effects are mediated to the particles in the MSSM in such a way that they do not distinguish particles in different generations ("flavor-blind") because they only depend on the gauge quantum numbers of the particles. Such a model was regarded difficult to construct in the past.[53] However, a breakthrough was made by Dine, Nelson and collaborators,[60] who started constructing models where the MSSM gauge interactions could indeed mediate the supersymmetry breaking effects, inducing postive scalar masses-squared and large enough gaugino masses (which used to be one of the most difficult things to achieve).[61] The original models had three independent sectors, one for supersymmetry breaking, one (the messenger sector) for mediation alone, and the last one the MSSM. Later models eliminated the messenger sector entirely[62] (see also reviews[63]).

Difficulty still remained how large enough gaugino masses can be generated in models where the sector of dynamical supersymmetry breaking couples to the MSSM fields only by Planck-scale suppressed interactions.[61] One could go around this problem by a clever choice of the quantum numbers for a gauge singlet field.[64] But it was not realized until recently that the gaugino masses are generated by superconformal anomaly.[33] This observation was confirmed and further generalized by other groups.[65] Randall and Sundrum further realized that one could even have scalar masses entirely from the superconformal

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anomaly if the sector of dynamical supersymmetry breaking and the MSSM particles are physically separated in the extra dimensions. The consequence was striking: the soft parameters were determined solely by the low-energy theory and did not depend on the physics at high energy scales at all. This makes it attractive as a solution to the problem of flavor-changing neutral currents, as the low-energy interactions of first and second generations are indeed nearly flavor-blind. Even though such models initially suffered from the problem that some of the scalars had negative mass-squared, simple fixes were proposed.[66] One can preserve the virtue of the anomaly mediation, namely ultraviolet insensitivity, and construct realistic models.

Finally a new idea called "gaugino mediation" came out lately.[34] This idea employs an extra dimension where the gauge fields propagate in the bulk. Supersymmetry is broken on a different brane and the MSSM fields learn the supersymmetry breaking effects by the MSSM gauge interactions, and hence solving the flavor-changing problem.

9 Conclusion

Supersymmetry is a well-motivated candidate for physics beyond the Standard Model. It would allow us to extrapolate the (supersymmetric version of the) Standard Model down to much shorter distances, giving us hope to connect the observables at TeV-scale experiments to parameters of the much more fundamental theories. Even though it has been extensively studied over two decades, many new aspects of supersymmetry have been uncovered in the last few years. We expect that research along this direction will continue to be fruitful. We, however, really need a clear-cut confirmation (or falsification) experimentally. The good news is that we expect it to be discovered, if nature did choose this direction, at the currently planned experiments.

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